A Study on teaching and learning problem-solving of the optimization problems in regional inequalities using GeoGebra (Study of Mathematical Software and Its Effective Use for Mathematics Education)

AUTHOR(S):

Choi, Kyeongsik

数理解析研究所講究録
第 1951 巻 2015 年 25-33

A Study on teaching and learning problem-solving of the optimization problems in regional inequalities using GeoGebra

Kyeongsik Choi
Sejong Global High School, Sejong, Korea

Abstract

This research is conducted to develop teaching and learning materials for problem-solving of the optimization problems in regional inequalities using GeoGebra. In the research, the three situations were developed according to the three steps which Polya suggested, induction and analogy, generalization and specialization. The materials developed were provided to students at the private interviews. At the interviews, students used the thinking way which Polya suggested for problem-solving. Plus, completing the questionnaire was required to the students after small group classes: Students preferred that infinitely many level curves were drawn on the plane instead of one level curve for their problem-solving. Also, GeoGebra was useful for them to observe infinitely many level curves and the change of $f(x,y)$ of a point on the plane.

1 Introduction

According to National Council of Teachers of Mathematics (2000), computer can make students express visually mathematical concepts, analyze data, and get broader mathematical experience. In National Education Curriculum 2009 of Korea, using technology tools is recommended for understanding concepts, principles of mathematics, enhancing problem-solving ability and evaluating complex calculation. For making math teachers use technology tools for teaching math in their classrooms, proper teaching and learning materials should be provided. Current math textbooks, however, have only one or two pages related with how to use technology tools for teaching or learning mathematics with them in the end of every chapters and most of Korean math teachers don't teach this content. On top of that, many materials developed previously cannot be used in these days, for current technology have been developed and overcome many defects of previous educational materials using technology. Teaching and learning materials using technology should be newly developed again with recent technology.

In this research, previous researches about developing teaching and learning materials for problem-solving of the optimization problem in regional inequalities were investigated and developing new materials with GeoGebra, which is a dynamic mathematics software, was conducted. In developing materials using technology, making students concentrate not how to use GeoGebra, but how to solve problems was considered. Students didn't learn how to use GeoGebra in detail. Instead, they tried to solve problems, and then used GeoGebra for using it as a tool for problem-solving.

2 Theoretical Background

Pólya (1954) thought Generalization, Specialization and analogy as important properties for problem-solving. Polya explained the three properties, Generalization, Specialization and Analogy in the problem-solving process in Figure 1. The right triangle of I is separated by the height from the hypotenuse in II. The relation between I and II is analogy. III in Figure 1 is the generalization of I, and II is the specialization of III. In this process, the strategies for problem-solving, generalization, specialization and analogy, can make students have much broader thought and get to know how to prove Pythagorean theorem.

While Pólya (1954) discussed the plausible reasoning, he proposed an example of the optimization of regional inequalities for the thought strategy.

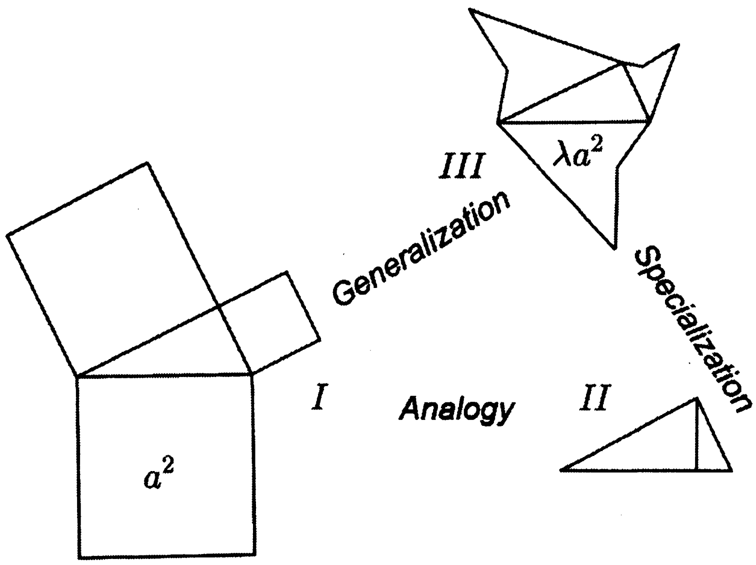


Figure 1: Generalization, Specialization, Analogy(Polya)

Given two points and a straight line, all in the same plane, both points on the same side of the line. On the given straight line, find a point from which the segment joining the two given points is seen under the greatest possible angle(Pólya, 1954).

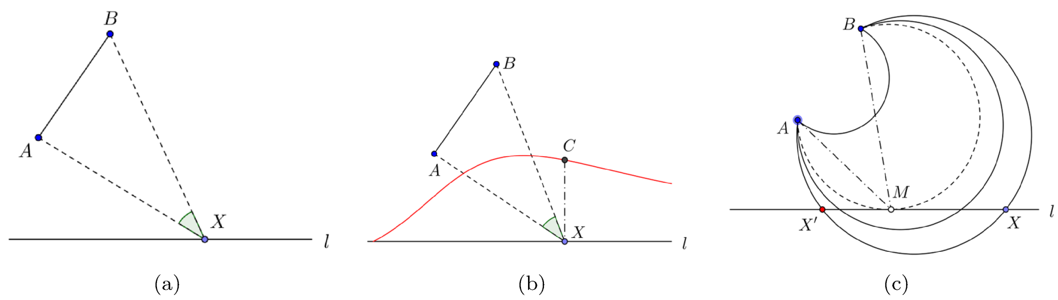


Figure 2: Tangent level curve for problem-solving the maximum problem

In Figure 2a, the problem means to find the position where the angle can be maximized looking segment AB at point X on line l. If there is a point C which has x coordinate as x coordinate of point X, y coordinate as the value of angle, the trace of point C seems like Figure 2b. Thus, the intersection point between a segment AB and a line l is the place where the size of angle is 0; if the point X moves to the right infinitely, the size of angle will converge 0 again. Therefore, there is points which can have the same angle value, for the two end of this ray is (almost) 0 and the size of angle is always positive and changes its size continuously. In Figure 2c, point X and X' have the same size of angle, because point A, B, X, X' are on the same circle. According the previous observation, the point for the maximum value of angle should be the only one. Therefore, point A, B, X should be on a circle and the circle should be a tangent line to the line l.

In the curriculum, there are some examples of the optimization with the concept of 'tangent level curve'. The first example is a multi-variable problem.

If point (x, y) is on $g(x, y) = x^2 + y^2 - 4 = 0$, evaluate the maximum and minimum of function

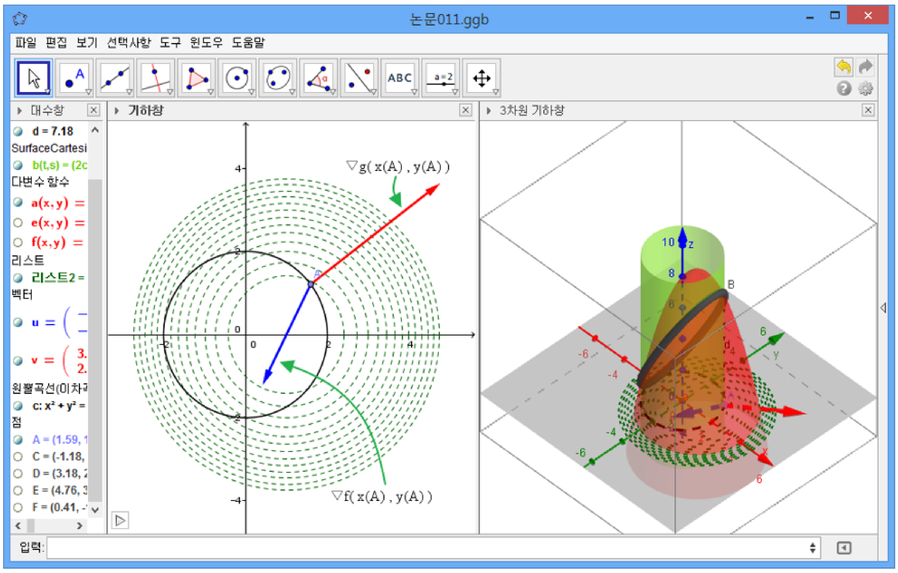


Figure 3: Lagrange's multiplier and tangent level curve

$f(x,y) = 9 - (x-1)^2 - y^2$.

Generally, Lagrange multiplier is used for solving this problem. Thus, the equations for solving this problem are as follows:

$$\begin{aligned} \nabla f(x_0, y_0) + \lambda \nabla g(x_0, y_0) &= 0 \\ g(x_0, y_0) &= 0 \end{aligned}$$

Figure 3 represents the proposed problem situation as 2D and 3D. The level curves in 2D figure shows the height from xy plane and the line is domain of function. Also, there are the two vectors of point A, ∇f and ∇g in Figure 3. $\nabla f(x_0, y_0) + \lambda \nabla g(x_0, y_0) = 0$ means that the two gradients are parallel and the level curve for getting the maximum or minimum should be tangent to the domain curve.

The second example for evaluating maximum and minimum is the optimization problem in regional inequalities as follows:

> In the intersection region of $x \geq 0$, $y \geq 0$, $2x+y \leq 6$, evaluate the maximum and the minimum of $2y - x$.

In Figure 4, there are many level curves on the Cartesian plane. According to the property of 'tangent level curve', the level curve should meet the only one point to the region.

3 Previous Researches

Seo (2009) pointed that many students learn the algorithmic method for solving the optimization problem in region inequalities and students have some difficulties for understanding the mathematical principles. Thus, Seo (2009) proposed that students should recognize the exact meaning of the problem:

1. evaluating the max-min value of function of points in domain
2. using the fact that domain is partitioned by the level curves for solving max-min problem

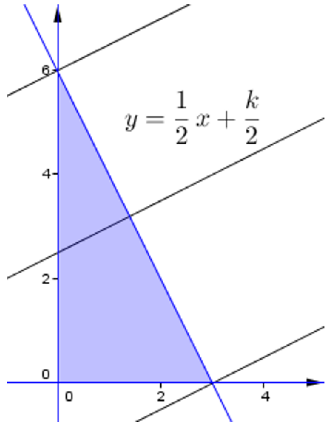
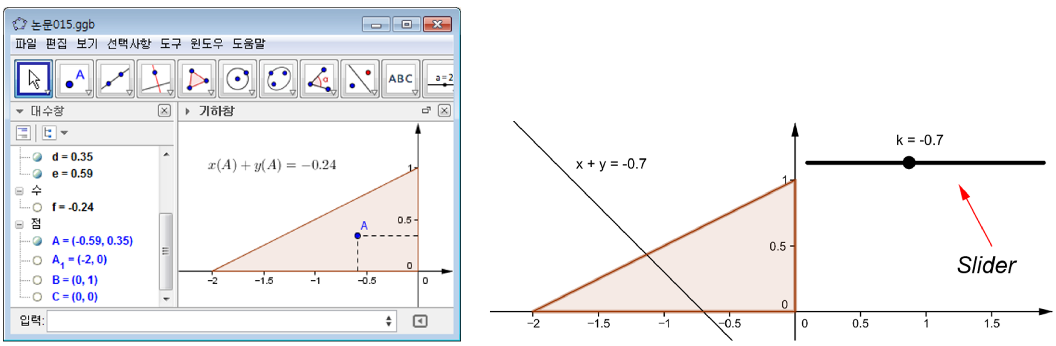


Figure 4: The optimization problem in regional inequalities and tangent level curve

Lee (2012) developed teaching and learning materials for helping students solve the optimization problem using GeoGebra's features, coordinate functions, function graph, slider tool. Lee applied the materials to her students and help them understand the meaning of $f(x, y) = k$, the relation of the variable k and the level curve $f(x, y) = k$.

Lee (2012), however, found that some unintentional pedagogical changes by the use of technology tools:

1. If students used coordinate function, they could get function value at every point easily. However, this made students recognize the necessity of using the level curves for solving the optimization problem(Figure 5a).
2. If students used the slider tool for solving the problem, students could get the value of the slider k, k-level curve. However, this made students forget the exact meaning of the problem, evaluating max-min value of the function at points, the method of calculating the point's coordinate to the function(Figure 5b).



(a) (b)

Figure 5: Lee(2012)'s teaching and learning materials for the optimization problems

When the technology tool is used for teaching and learning materials, the alternative way should be resolved. In this research, teaching and learning materials using GeoGebra without the unintentional change of pedagogical object. Also, GeoGebra support students' inductive inference.

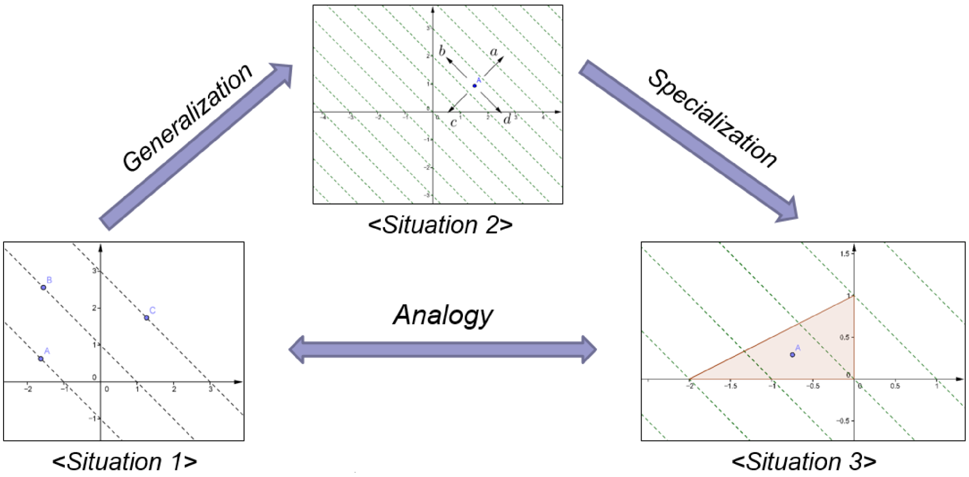


Figure 6: The process for problem-solving the optimization problem(I)

4 Developing teaching and learning materials

In this research, the materials were developed for solving the problem as follows:

> If a point (x, y) is in the intersection of three region: $x \le 0$, $y \ge 0$, $x - 2y \ge -2$. Evaluate the maximum and minimum of $x + y$.

For solving this problem, the first step is to understand the concept of the level curves(Lee, 2012). In general math textbooks, k-level curve for $f(x, y) = x+y$ and the method of changing k-value is proposed. In this research, the materials were developed using the three properties: generalization, specialization and analogy. Situation I provides some questions, which have activities for understanding the properties of a point on a level curve with GeoGebra's coordinate functions, for students to make an inference about the concept of level curves inductively. Situation 2, the generalization of situation 1, provide a picture and some questions to explore the changing of value along the movement of a point. From situation 2, students may understand that the plane is partitioned by infinitely many level curves(lines). Situation 3, the specialization of situation 2, provide activities about getting the maximum and minimum values of $f(x, y)$ in regional inequalities(Figure 6, Figure 7).

5 Discussions

In this research, three interviewees were participated and they had learned the problem-solving the optimization problem with a teacher.

All students answered the questions of situation 1(Figure 8), but a student didn't know the exact reason for the answer. Teacher asked of the question again to the student, and the student tried to think of the problem again.

1a Teacher : Looking point A, B, C on the lines, how much is the sum of x and y coordinate value of point B?

1b Teacher : Can you guess the value?

2 S1 : No, it is not accurate.

3 Teacher : No. It is not accurate. By the way, you do something?

4 S1 : Um, are you asking the coordinate of this point?

.

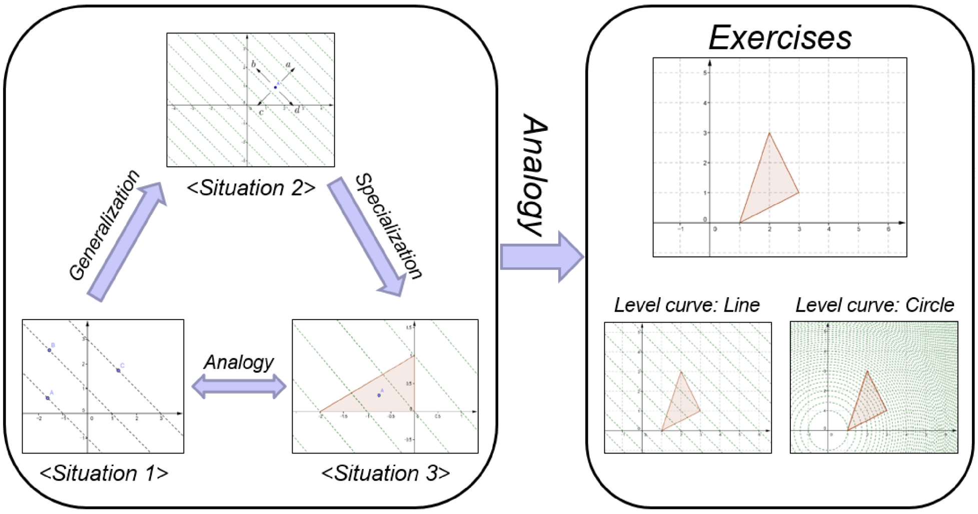


Figure 7: The process for problem-solving the optimization problem(II)

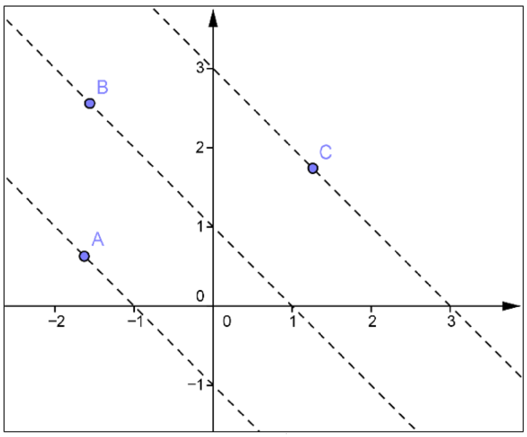


Figure 8: Situation 1

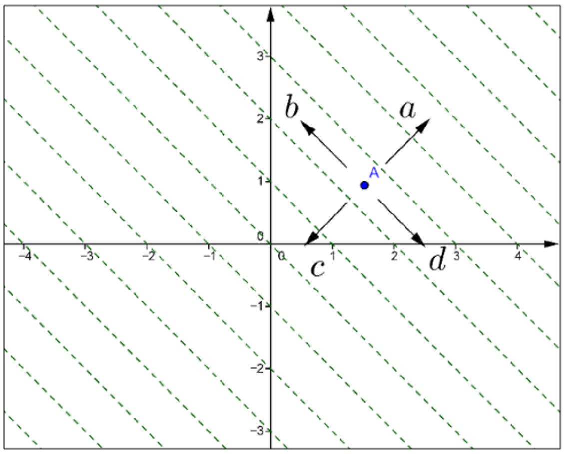


Figure 9: Situation 2

5 Teacher : Yes, x coordinate and y coordinate. How much is the sum of x and y coordinate value of point A?

6 S1 : This question asks of x and y of this equation.

7 Teacher : No. This question means x coordinate and y coordinate of point A.

8 S1 : Ah...

9 Teacher : Isn't it?

10 S1 : Ah, it is. I misunderstood....

Situation 2 is the generalized one of Situation 1. In Situation 2, students could expect the change of $f(x, y)$'s value according to a point which could move freely on the Cartesian plane. At 44b, S1 could guess the value of $f(x, y)$ according to the point's coordinate using level curves of Situation 1.

41 Teacher : The point, which direction will make the value to increase? The first question..

42a S1 : Direction a will make the value incresing.

43 Teacher : Uh, Why?

44a S1 : Um, just before, just a minute. Can I see the previous thing(Situation 1)?

44b S1 : According to this, here is -1. When $x + y$ is 1, here. When $x + y$ is 3, here.

44c S1 : Therefore, they is moving to upward as the number is growing bigger.

44d S1 : So, if $x + y$ is growing bigger, it(point) will move this direction.

Situation 3 is the specialized one of Situation 2. In Situation 3, students could get to know the max-min values of $f(x, y)$ with the point in the region provided. At 47, S1 could expect 49a using the result which was observed in Situation 2 like 47.

45 S1 : If it(point) moves this direction, the value won't be changed.

46 Teacher : Um.

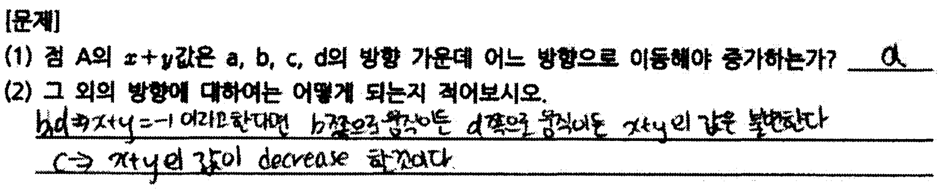

[문제]
(1) 점 A의 $x+y$값은 a, b, c, d의 방향 가운데 어느 방향으로 이동해야 증가하는가? a
(2) 그 외의 방향에 대하여는 어떻게 되는지 적어보시오.
b,d ⇒ x+y=−1 이라고 한다면 b쪽으로 움직이든 d쪽으로 움직이든 x+y의 값은 불변한다
c ⇒ x+y의 값이 decrease 할것이다

Figure 10: Worksheet(Situation 2)

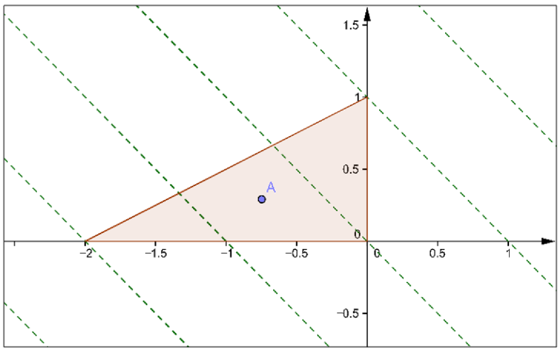
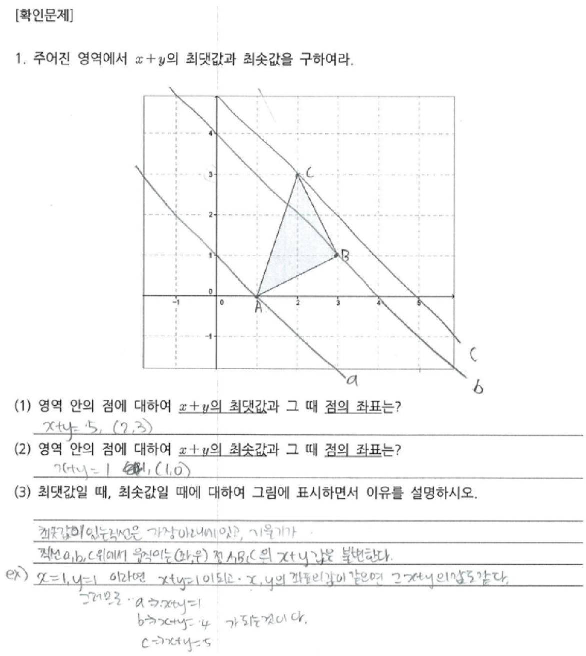


Figure 11: Situation 3


[확인문제]

1. 주어진 영역에서 $x+y$의 최댓값과 최솟값을 구하여라.

(1) 영역 안의 점에 대하여 $x+y$의 최댓값과 그 때 점의 좌표는?
x+y=5, (2,3)
(2) 영역 안의 점에 대하여 $x+y$의 최솟값과 그 때 점의 좌표는?
x+y=1, (1,0)
(3) 최댓값일 때, 최솟값일 때에 대하여 그림에 표시하면서 이유를 설명하시오.
최댓값이 있는 직선은 가장 아래에 있고, 기울기가
직선 a,b,c 위에서 움직이는(좌,우) 점 A,B,C의 x+y값은 불변한다.
ex) x=1, y=1 이라면 x+y=1 이되고 · x, y의 좌표의 값이 같으면 그 x+y의 값도 같다.
그러므로 ∴ a ⇒ x+y=1
b ⇒ x+y=4 가 되는 것이다.
c ⇒ x+y=5

Figure 12: Exercises

47 S1 : By the way, just before you said... I said just before, this way makes the value increasing, that way makes the value ... decreasing.

48 Teacher : Decreasing.

49a S1 : So, the point can move only in this figure, the most downward part is this.

49b S1 : So, this value is the lowest, the top part of this is point A.

Solving exercises, students could expect the change of $x+y$'s value using level curves as the auxiliary tool for thinking by themselves with GeoGebra.

At the questionnaire survey, students answered that many level curves drawn on the Cartesian plane were more helpful to solve the problem than the only one level curve on the plane. Moreover, GeoGebra was useful to draw graph of function and effective to solve problems and understand mathematical principles.

6 Result

For teaching mathematics using technology tool in school, topics which can be helpful for students to solve problems should be chosen. Teachers also should guide to get proper mathematics concepts and enhance problem-solving ability.

The objective of this research is to develop teaching and learning materials which can be used for teaching problem-solving using GeoGebra. Especially, the three situations along the three strategies, generalization, specialization and analogy, were developed and applied to a few students for teaching problem-solving of the optimization problem in regional inequalities. Students in this research understood the idea of level curves for solving the optimization problem and used GeoGebra to justify their hypothesis.

It is worth to be expected that researches related with this research should be investigated about problem-solving with technology tool.

References

Lee, S. (2012). A Study on Learning and Teaching of the Optimization Problems in Regional Inequalities Using GeoGebra. Master's thesis, Korea National University of Education, Korea.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, Va.:NCTM.

Pólya, G. (1954). *Mathematics and plausible reasoning: Induction and analogy in mathematics*, volume 1. Princeton University Press.

Seo, H. (2009). A study about teaching in the area of Inequality : A problem of maximum and minimum. Master's thesis, Seoul National University, Korea.